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Flows of cars and neural spikes enhanced by colored noise

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Abstract

The problem of interaction between cars at road junctions has been studied for a long time. It is well-known that the interaction generally decreases the traffic efficiency and that randomness in the car flow can help cross-junction traffic efficiency. We show that proper long-range correlations in the flow noise, related to $1/f$-like noises, provide superior traffic properties as compared to Poissonian or periodic car traffic. Moreover, a stochastic resonance phenomenon sensitive to the shape of the spectrum occurs. By a small modification, the model can be made relevant for neuronal spike traffic. Spike trains generated by Gaussian $1/f$ noise show superior spike traffic properties and efficiency of information transfer. © 2000 Published by Elsevier Science B.V. All rights reserved.

1. Introduction

Noise-assisted signal transfer in nonlinear systems is a hot topic of today's physics of complex systems. Important examples are stochastic resonance (SR) phenomena in various physical and biological systems [1–12], which have recently attracted a strong interest. Numerous non-linear systems exhibit SR and are often called 'stochastic resonators' [2,5]. A proper tuning of the input noise intensity is needed to optimize the transfer of a deterministic signal through the stochastic resonator [1–12]. One of the most important research directions of today's SR research is the investigation of signal transfer in chemical [9] and biological [1,4] systems, however, there are many other fields such as threshold devices [2,5,8], SQUIDs ([11] and references therein), etc., where SR has been studied. The phenomenon discussed in the present paper is a new type of stochastic resonance effect, where the optimal tuning concerns the *shape* of the power density spectrum rather than the *intensity* of the input noise.

Probably, every car driver has experienced a beneficial effect of randomness. In a certain crowded traffic situation, it is not possible to get into a major road from a lower priority road, if the cars are passing the junction periodically. The gaps between subsequent cars on the main road are not large enough for a waiting car to get into that gap and to accelerate to the required speed. Only a larger gap, which usually occurs randomly in the car flow,

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makes it possible to enter. Therefore, noise can be beneficial for the traffic in certain cases. Similar problems of car traffic including the problem of the gap have been extensively studied in the literature in the last three decades [13–15]. However, the questions of an optimal noise and of a possible stochastic resonance have not been addressed yet according to our literature search. In this Letter, we shall investigate the effect of various noise processes on the traffic and we shall show that there is an apparent stochastic resonance phenomenon, which concerns the shape of the noise spectrum.

2. The car traffic model

The problem investigated in this Letter can be visualized as follows. There is a main road with a junction, where, for the sake of simplicity, one-way traffic is assumed (see Fig. 1). A car waiting at the junction (2) can enter into the road if the gap size between two successive cars is greater than G_0. A busy traffic is assumed with a large number of cars waiting at the junction. If the size G_i of the ith gap is larger than G_0, the number of cars which enter in that gap is:

$$N_i = \mathrm{Integer}(G_i/G_0) \tag{1}$$

We are interested in the mean frequency $\nu_{1,3}$ of cars going from 1 to 3 (main road traffic rate) and the mean frequency $\nu_{2,3}$ of cars going from 2 to 3. We characterize the overall traffic efficiency ν of the system by the geometric mean $\nu = (\nu_{1,3}\nu_{2,3})^{0.5}$ of these two frequencies. In this way, the quantity ν has small values for those traffic patterns when the $\nu_{1,3}$ and/or the $\nu_{2,3}$ are/is poor. With this model, we have investigated three different classes of point processes controlling the $\nu_{1,3}$. Periodic traffic, a case with no noise; Poisson process, a case with noise but without memory effects; and zero crossing events of colored noise processes with $1/f^k$ spectrum, which are cases with long-range memory depending on the value of the spectral exponent k.

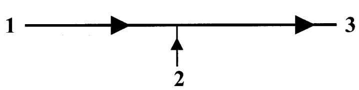

Fig. 1. Outline of a simple traffic system. Road 2 has a lower priority. A car waiting at the crossing on road 2 can get into road 1 only when there is a sufficiently large gap ($=G_0$) in the car flow on road 1.

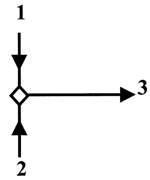

Fig. 2. Outline of a simple neural traffic system. Spikes coming from 1 and 2 have the same priority.

3. The neural spike traffic model

With a simple modification, which is a simplification, the car traffic model becomes relevant for neural spike traffic (see Fig. 2). In this model, roads (channels) 1 and 2 have equal priority. The neuron transfers the spike coming from channel 1 or 2, if the time since the last transferred spike is greater than G_0, where G_0 represents the refractory time of the neuronal junction. In this model, the frequency of transferred spikes depends on the statistics of spike generation; moreover, the spikes can be lost. Both the output spike frequency and the probability of spike loss influence the efficiency of information transfer, therefore, we ask the following question: what kind of spike statistics provides the best efficiency of information transfer?

4. Results on the car traffic model

First, consider the case of periodic traffic. It is obvious that the upper limit of $\nu_{2,3}$ corresponding to $\nu_{1,3}$ equal zero is:

$$\nu_{2,3\max}^{\mathrm{peri}} = 1/G_0 . \tag{2}$$

From Eq. (1) it follows that, for arbitrary $\nu_{1,3}$, the $\nu_{2,3}$ obeys:

$$\nu_{2,3}^{\mathrm{peri}} = 1/G_0 - \nu_{1,3}^{\mathrm{peri}} \text{ when } \nu_{1,3}^{\mathrm{peri}} \le 1/G_0 , \tag{3a}$$

$$\nu_{2,3}^{\mathrm{peri}} = 0 \text{ when } \nu_{1,3}^{\mathrm{peri}} \ge 1/G_0 . \tag{3b}$$

Consider now Poisson generated traffic [13,14]. Here the time moments when cars on the main road pass the junction are generated by a Poisson process with rate $\nu_{1,3}^{\text{pois}}$. The probability to find exactly M events (cars passing the junction) during time interval τ is described by:

$$P_M(t) = \frac{\left(\nu_{1,3}^{\text{pois}}\tau\right)^M}{M!}\exp\left(-\nu_{1,3}^{\text{pois}}\tau\right), \tag{4}$$

To calculate the relationship between mean rates $\nu_{1,3}$ and $\nu_{2,3}$, we define the moment of a car passing the junction as $t_0 = 0$, and introduce two (positive) observation times t_1 and t_2, such that $t_1 < t_2$. From Eq. (4) it follows that the probability to have events (one or more) during time interval $t_1 < t < t_2$ if there were no events at $t < t_1$ is:

$$\begin{aligned} P_{M\geq 1,t_1,t_2} &= P_0(t_1) - P_0(t_2) \\ &= \exp\left(-\nu_{1,3}^{\text{pois}}t_1\right) - \exp\left(-\nu_{1,3}^{\text{pois}}t_2\right). \end{aligned} \tag{5}$$

In our model this equation gives the probability of accommodating exactly one car from the junction if we put $t_1 = G_0$ and $t_2 = 2G_0$ (one interval of duration G_0 is 'clean', but not two). The probability of accommodating exactly two cars is obtained from Eq. (5) if $t_1 = 2G_0$ and $t_2 = 3G_0$ (two intervals of duration G_0 are clean, but not three). And so on. The mean number of cars accommodated from the junction per single interval between cars in the main road is given by the sum of these probabilities weighed by the corresponding car numbers:

$$\begin{aligned} \langle N\rangle &= \sum_{n=1}^{\infty} n\left[\exp\left(-\nu_{1,3}^{\text{pois}}G_0 n\right)\right. \\ &\qquad \left. -\exp\left(-\nu_{1,3}^{\text{pois}}G_0(n+1)\right)\right] \\ &= \frac{\exp\left(-\nu_{1,3}^{\text{pois}}G_0\right)}{1-\exp\left(-\nu_{1,3}^{\text{pois}}G_0\right)}. \end{aligned} \tag{6}$$

Thus, the average rate of entering from the junction is:

$$\nu_{2,3}^{\text{pois}} = \nu_{1,3}^{\text{pois}} \quad \langle N\rangle = \frac{\nu_{1,3}^{\text{pois}}\exp\left(-\nu_{1,3}^{\text{pois}}G_0\right)}{1-\exp\left(-\nu_{1,3}^{\text{pois}}G_0\right)}. \tag{7}$$

Finally, consider the case when the point process describing the car occurrence on the main road is generated by the zero crossing events of a Gaussian $1/f^k$ noise. When $k > 0$, this noise has a long-range memory. Due to the experimental evidence of occurrence of $1/f^k$-like noise processes in car traffic [16], it is tempting to apply this kind of noises $(0 < k < 2)$ to generate the car occurrence. The mean zero crossing rate of a Gaussian noise process is described by the Rice formula [17,18]:

$$\nu_{1,3}^{\text{color}} = 2\frac{\sqrt{\int_0^\infty f^2 S(f)\,\mathrm{d}f}}{\sqrt{\int_0^\infty S(f)\,\mathrm{d}f}}, \tag{8}$$

where f is the frequency and $S(f)$ is the power density spectrum of the noise. The value of $\nu_{2,3}^{\text{color}}$ cannot be calculated analytically because the time distribution of the zero crossing events has been an unsolved problem since 1945 [17–20].

To compare the three different cases of traffic processes, we carried out computer simulations, which are described below. The length of the simulation and the point processes describing the car occurrence on the main road were 32768 (2^{15}) and the minimal gap size G_0 was 20. In comparison with a practical highway traffic situation, where the mean distance between the cars is 100 m, the total process length corresponds to the main traffic road of 160 km. In Fig. 3, $\nu_{2,3}$ versus $\nu_{1,3}$ is shown. The results for Poissonian traffic turned out to be in excellent agreement with the predictions of Eq. (7) and can be regarded as a test of the simulation accuracy. The frequency range of integration relevant in Eq. (8) was determined by the simulation length, so the colored noise traffic was solely controlled by the spectral exponent k. For the results shown in Fig. 3, the range 0–2 was used for k. It is obvious that the

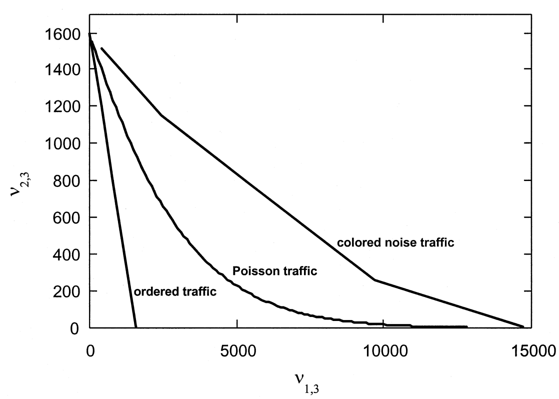

Fig. 3. Superiority of colored ($1/f^k$ noise generated) car traffic over Poissonian and periodic traffic processes.

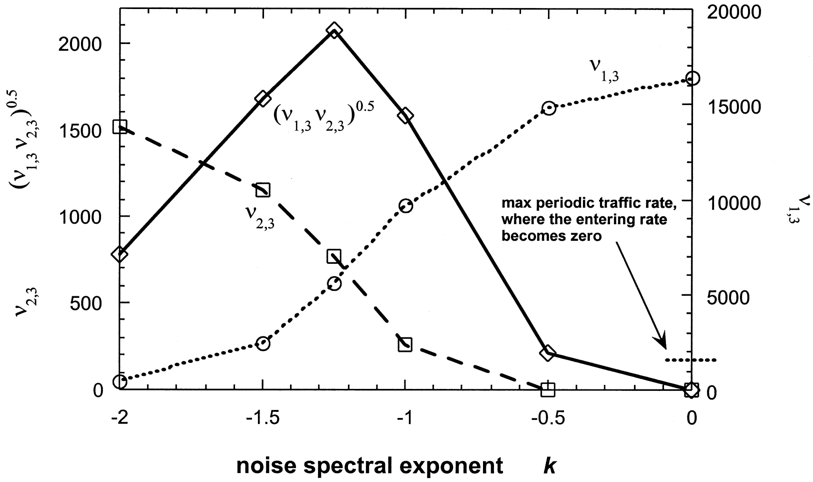

Fig. 4. Stochastic resonance peak in the car traffic efficiency $\nu = (\nu_{1,3}\ \nu_{2,3})^{0.5}$ (solid line) versus the spectral exponent of the colored noise traffic.

periodic traffic gives the smallest $\nu_{2,3}$ and the colored noise traffic gives the largest $\nu_{2,3}$, especially for medium and high $\nu_{1,3}$. It is also apparent that the best compromise between the $\nu_{1,3}$ and $\nu_{2,3}$ is provided by the colored noise traffic while the worst one corresponds to the periodic traffic.

In the case of the colored noise generated traffic, the most interesting result is a new kind of SR, *spectral stochastic resonance* (SSR), which demonstrates the existence of an optimal spectral shape (k) for the highest traffic efficiency ν, as shown in Fig. 4. This new kind of stochastic resonance is similar to the classical effects in the sense that the noise driving is needed to get the optimal performance of the system. On the other hand, instead of the noise intensity, the spectral shape (as described by k) is the SR tuning parameter, which optimizes the performance. Note, that usually, the spectral shape is more related to resonance effects in physics than intensity.

Musha and Higuchi [16] reported a $1/f$-like noise in the traffic current of cars on highways, so, it is particularly interesting that the optimal traffic in our model is also found around $k = 1$.

5. Results on the neural spike traffic model

The neural traffic model also shows nontrivial features when driven by $1/f^k$ noise generated spikes. This time, the introduction of the overall traffic efficiency, as the geometrical mean of the two traffic rates, is not necessary. The mean frequency of the outgoing spikes is a good measure of the variations in the upper limit of information transfer rate through the system. Computer simulations were done in a similar way and with similar conditions as described above. The upper frequency cut-off of the spectrum was chosen to be 6000 (compare with the sampling frequency of 32000), which is equivalent to a refractory time of 5.3, characterizing spike statistics of the sources of spikes. The refractory time of the junction–neuron, which corresponds to the minimal gap size in the car traffic model, was chosen 10. In Fig. 5, the mean rate of transferred spikes and the proba-

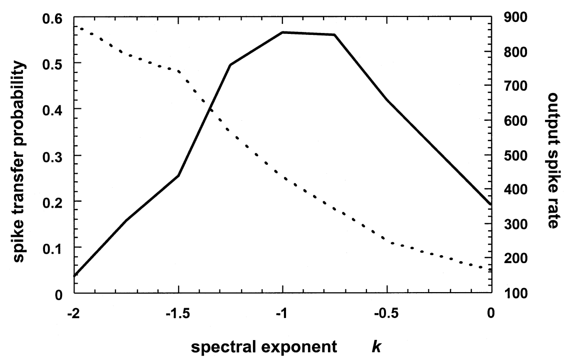

Fig. 5. Stochastic resonance peak in the frequency of transferred neural spikes (solid line) versus the spectral exponent of the colored noise traffic. The dashed line shows the probability of spike loss.

bility of spike transfer are shown. The rate of transferred spikes characterizes the highest meaningful bandwidth of information transfer, due to Shannon's sampling theory. Therefore, this quantity is directly related to the information transfer rate.

Here we can also observe a well-pronounced SSR around $k = 1$. This fact is in an interesting coincidence with the general occurrence of $1/f$-like noise phenomena frequently reported in neural activity [21–24]. The other quantity, the spike transfer probability characterizes the phenomenon of spike loss. The actual rate of information transfer depends on both quantities as well as on the unknown way of neural coding. The spike loss is the smallest at $1/f^2$ noise (Brownian motion) generated spike train, however, in this case, the widest meaningful bandwidth of information transfer is one order of magnitude less than at the case of $1/f$ noise. The $1/f$ noise case provides the highest spike propagation rate, though, with some compromise in the accuracy of transfer.

6. Conclusion

It is shown that traffic flows generated by $1/f^k$ noises have superior properties over Poissonian and periodic cases. The best properties are achieved around $k = 1$. This fact is in an intriguing coincidence with the general occurrence of $1/f$-like noise phenomena in neural activity and highway car traffic.

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